Lecture 4

The Fast Fourier Transform (Mario Garrido)

Sorting on GPU

OpenGL interoperability

Lecture questions

1) What is the challenge in parallizing the FFT?

2) In what way does bitonic sort fit the GPU better than many other sorting algorithms?

3) What is the advantage of using CUDA OpenGL interoperability?

Sorting on GPUs

Revisiting some algorithms from lecture 6:

Some not-so-good sorting approaches

Bitonic sort

QuickSort

Concurrent kernels and recursion

Adapt to parallel algorithms

Many sorting algorithms are highly sequential

Suitable for parallel implementation?

- Data driven execution
- Data independent execution

Data driven execution

Computing pattern depends on data

Usually harder to parallellize!

Example: QuickSort.

Data independent execution

Known computing pattern

Easier to parallellize - always the same plan

Example: Bitonic sort

Bubble sort

Loop through data, compare neighbors

Extremely sequential

Inefficient

Parallel version: Bubble sort with odd-even transposition method

Compare all items pairwise

Two phases, ”odd phase” and ”even phase” (shifted one step”

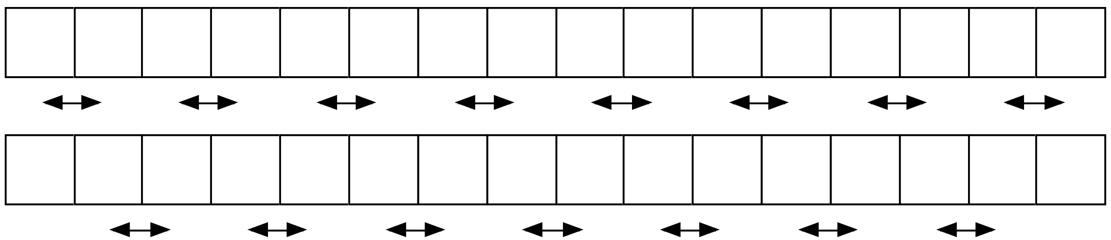
Bubble sort, parallel version

Bubble sort with odd-even transposition method

Compare all items pairwise

Two phases, ”odd phase” and ”even phase” (shifted one step”

Fully sorted after n phases



Even phase

Odd phase

$O(n^2)$

Suitable for GPU?

Not as bad as it seems at first look:

• Data independent

• Excellent locality

• Pretty good possibilities to use shared memory (but with some costly transfers at edges between blocks). Thus close to optimal in global memory transfers.

• But certainly not optimal at very large sizes

”Better” algorithms don’t necessary beat this all that easily!

Rank sort (lab 6)

Count number of items that are smaller

Easy to parallelize:

- **One thread per item**
- **Loop through entire data**
- **Store in index decided from count of number of smaller items.**

Suitable for GPU?

Again, not as bad as it seems at first look:

- **Data independent**
- **Excellent locality - especially good for broadcasting (e.g. constant memory). Also suitable for shared memory.**
- **Again, $O(n^2)$: Will grow at very large sizes**

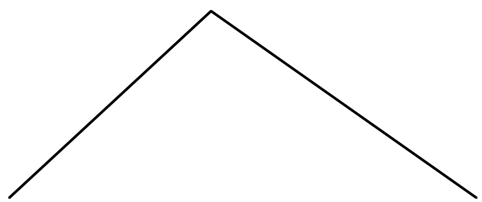
Two bad ones that are not quite as bad as they seem.

N parallel iterations may beat NlogN sequential ones!

Bitonic sort

(As described in lecture 6)

Bitonic set: Two monotonic parts in different direction.



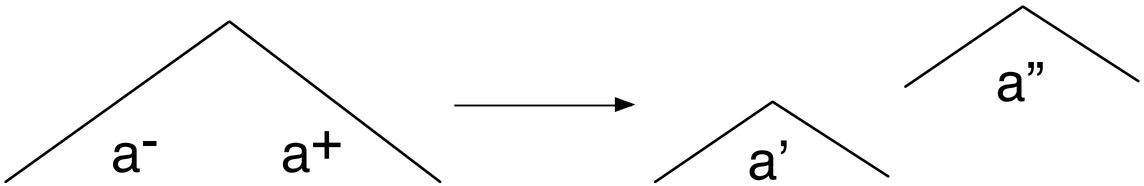
Bitonic sort

(According to Batcher:) Let a be a bitonic set with a maximum at k, consisting of two monotonic parts, one increasing, a^- (from item 1 to k) and one decreasing, a^+ (k+1 to n)

Then two new sets can be constructed as

$$a' = \min(a_1, a_{k+1}), \min(a_2, a_{k+2})\ldots$$
$$a'' = \max(a_1, a_{k+1}), \min(a_2, a_{k+2})\ldots$$

These two sets are also bitonic and max(a’) ≤ min(a”)!

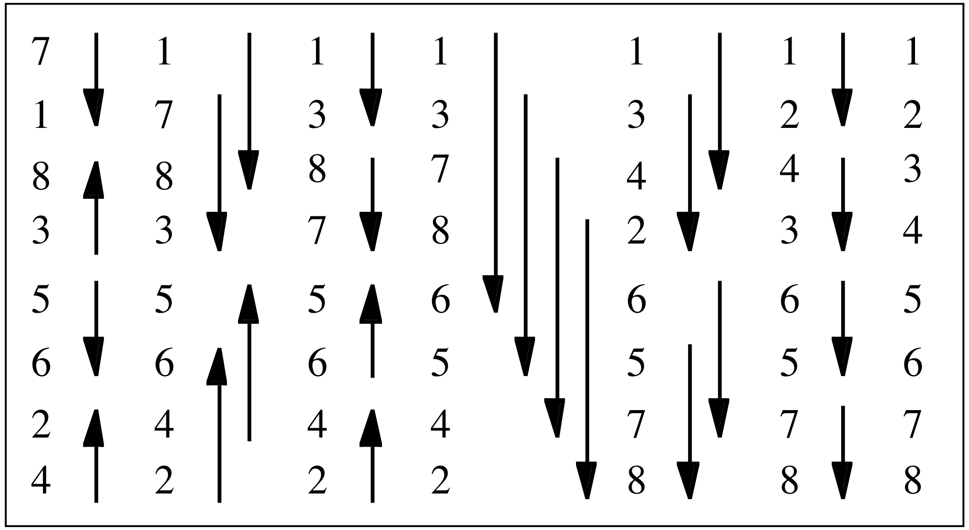


Bitonic sort by divide-and-conquer

Bitonic sort works on a bitonic sequence: partially sorted

The parts must be sorted. Sort them by bitonic sort!

Bitonic sort example



Bitonic sort of smaller part

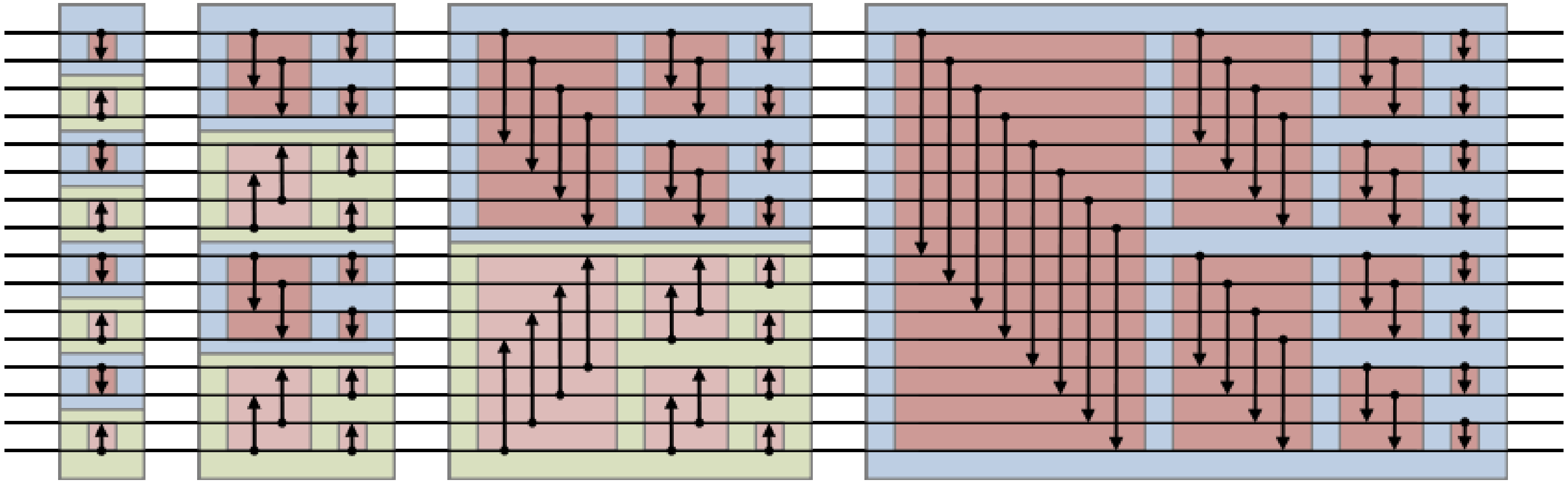
Reverse parts (bitonic merge)

Bitonic sort of main part

Reverse parts (bitonic merge)

Bigger example

The problem scales nicely, uniformly



More stages gives longer stages

(Image from Wikipedia)

Bitonic sort

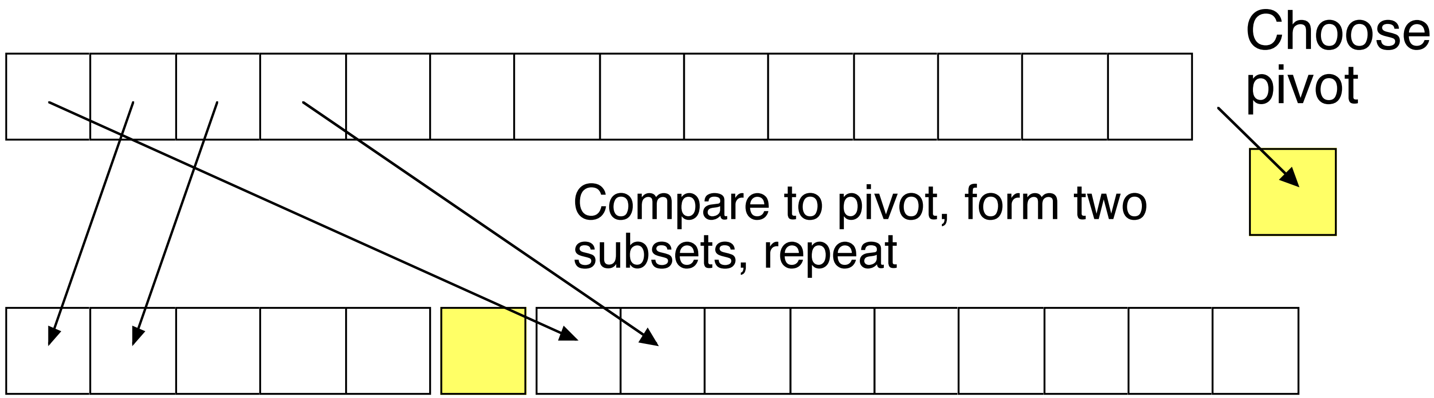
- Data independent, no worst case
- Fast: $O(n \cdot \log^2 n)$ (Why?)
- Good locality in some parts

but

- Big leaps in addressing for some parts

QuickSort

Very popular algorithm for sequential implementation



Data driven, data dependent reorganization, non-uniform

Fancy name - nobody expect QuickSort to be nothing but optimal

QuickSort is

Fast: O(n·logn) in typical cases

$O(n^2)$ in the worst case

Data driven, data dependent reorganization, non-uniform

Fancy name - nobody expects QuickSort to be nothing but optimal

QuickSort on GPU

Initially ignored as impractical

CUDA implementations exist

Data driven approaches increasingly suitable as GPUs become more flexible

Parallel QuickSort

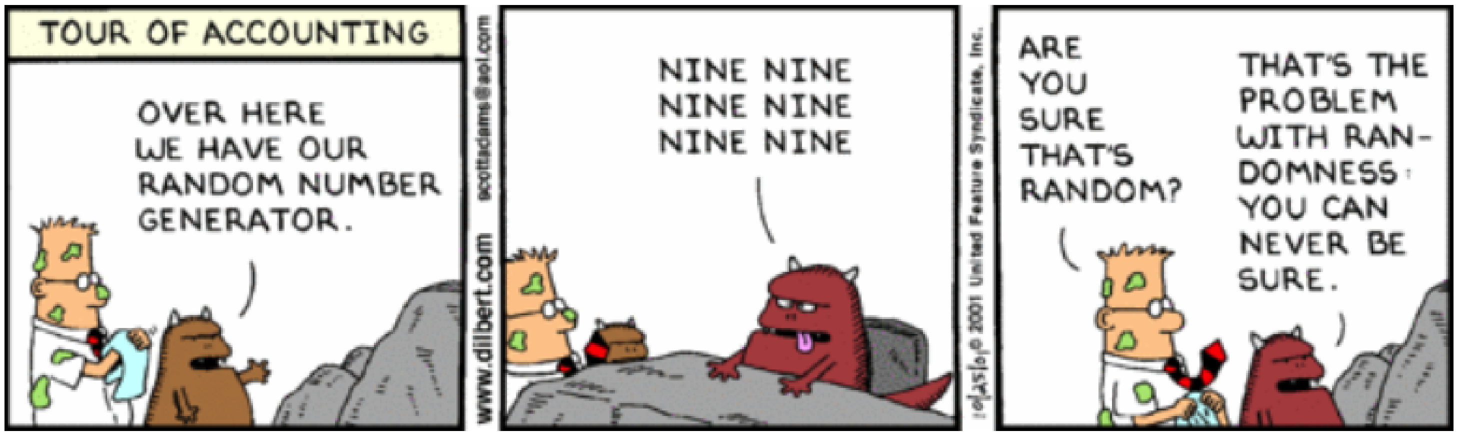
Several stages to consider:

- Pivot selection. Usually just grab one.
- Comparisons
- Partitioning
- Concatenate result

Pivot selection

If we could always pick a pivot that splits the data in half…

but you can’t do that without sorting! But how about a random one?



There is a worst case caused by bad pivots. Live with it!

Comparisons

Easy to parallelize

One thread per comparison not unreasonable!
(GPUs don't have a problem with many threads!)

No problem!

Partitioning

The big problem!

Sequential partitioning: Bad!

Parallel partitioning 1: Atomic fetch & increment.
(GPUs have atomics!)

Parallel partitioning 2: Divide and conquer

Recursion

GPUs can’t do recusion efficiently… or can they?

New in Kepler: *Concurrent kernels*

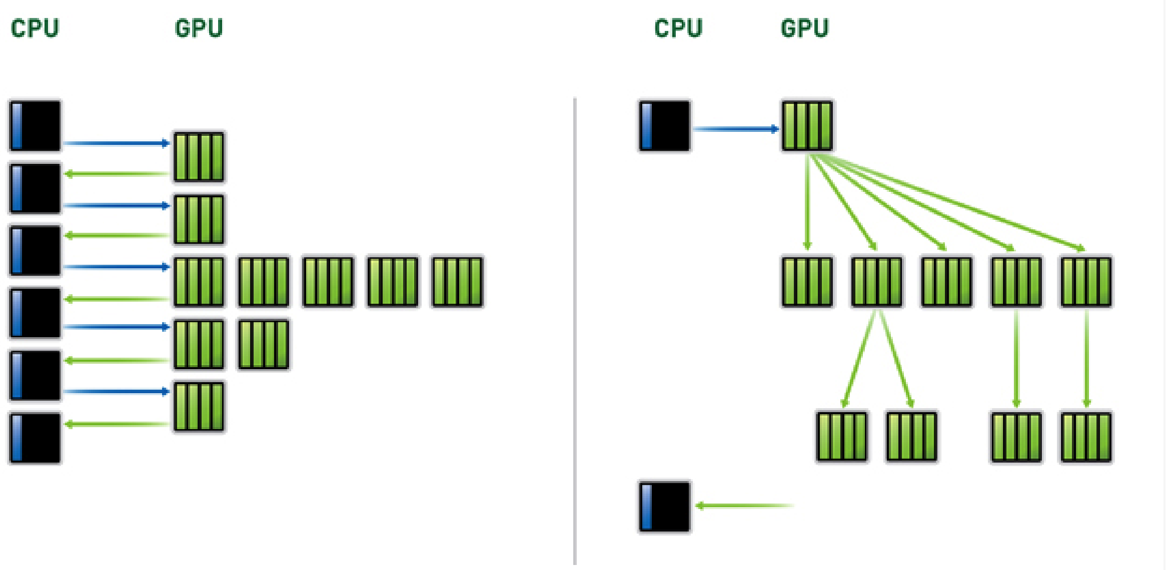
Not only a matter of launching kernels from CPU!

A kernel can spawn new kernels!

Do recursion by spawning new kernels!

Concurrent kernels, Dynamic Parallelism

Less work for the CPU to manage the computation.



Recursion can look like this:

```
__global__ void quicksort(int *data, int left, int right)
{
    int nleft, nright;
    cudaStream_t s1, s2;

    // Partitions data based on pivot of first element.
    // Returns counts in nleft & nright
    partition(data+left, data+right, data[left], nleft, nright);

    // If a sub-array needs sorting, launch a new grid for it.
    // Note use of streams to get concurrency between sub-sorts
    if(left < nright) {
        cudaStreamCreateWithFlags(&s1, cudaStreamNonBlocking);
        quicksort<<< ..., s1 >>>(data, left, nright);
    }
    if(nleft < right) {
        cudaStreamCreateWithFlags(&s2, cudaStreamNonBlocking);
        quicksort<<< ..., s2 >>>(data, nleft, right);
    }
}

__host__ void launch_quicksort(int *data, int count)
{
    quicksort<<< ... >>>(data, 0, count-1);
}
```

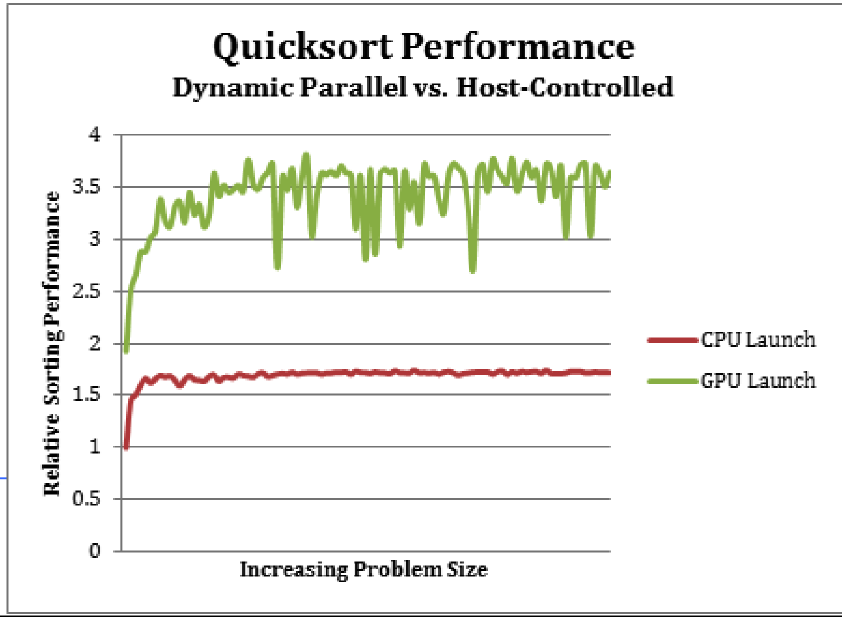
But… does this really do a good job on partitioning?

Source: http://blogs.nvidia.com/blog/2012/09/12/how-tesla-k20-speeds-up-quicksort-a-familiar-comp-sci-code/

Advantages

- Less work for CPU
- Less synchronizing (from CPU side)
- Easier programming!

They claim it matters this much (but your milage will vary)



Recursive CUDA kernels, a promising improvement

Big change in GPU computing?

Southfork has GPUs that support it.